§1

1.1 (i) $\alpha = (1783)(256)(4)$

(ii) $\beta = (12436)(578)$

(iii) $\alpha^{-1} = (1387)(265)(4)$

(iv) $\beta^{-1} = (16342)(587)$

(v) $\alpha\beta = (18643275)$

(vi) $\beta\alpha = (15867324)$

(vii) $\alpha^{-1}\beta^{-1}\alpha\beta = (1358)(2)(47)(6)$.

1.2 (i) TRUE as (12345) and (34512) have the same "cyclic ordering".

(ii) FALSE because (1234) maps $1 \mapsto 2$ and (1342) maps $1 \mapsto 3$. $\therefore$ $(1234) \neq (1342)$.

(iii) TRUE because (4) and (5) are just the identity permutation.

(iv) TRUE (1) and (5) are both the identity permutation.

(v) FALSE. In fact $(123)(45) = (45)(312)$.

(Note disjoint cycles commute.)

1.3 (i) There are three right cosets of H in G:-

$$H(1) = H(13) = \{(1), (13)\} \ (=H)$$
$$H(12) = H(132) = \{(12), (132)\}$$
$$H(23) = H(123) = \{(23), (123)\}$$

(ii) There are two right cosets of H in G:-

$$H(1) = H(123) = H(132) = \{(1), (123), (132)\} \ (=H)$$
$$H(12) = H(13) = H(23) = \{(12), (13), (23)\}$$

(iii) There are four right cosets of H in G:-

$$\{(1), (234), (243), (23), (24), (34)\} \ (=H)$$
$$\{(12), (1234), (1243), (123), (124), (12)(34)\}$$
$$\{(13), (1342), (1324), (132), (13)(24), (134)\}$$
$$\{(14), (1423), (1432), (14)(23), (142), (143)\}$$

1.4 (i) 10 (ii) 3 (iii) 7

1.5 σ has order r

1.6 For $m \in \mathbb{N}$, since disjoint cycles commute

$$\sigma^m = \sigma_1^m \sigma_2^m \cdots \sigma_t^m .$$

Also the $\sigma_1^m, \sigma_2^m, \ldots, \sigma_t^m$ are disjoint cycles

$$\sigma^m = (1) \iff \sigma_i^m = (1) \text{ for all } i = 1, \ldots, t.$$

Since $\sigma_i^m = (1) \iff m$ is divisible by the length of σ_i, the order of σ is the least common multiple of the lengths of $\sigma_1, \sigma_2, \ldots, \sigma_t$.

1.7 (i) 12 (ii) 15 (iii) 12 (iv) 15

(v) 8 (vi) 8 (vii) 4