Dec 2015

1) i) Def^n 1.24 - A matrix E is in (row) echelon form it it has the following two properties:

(i) The zero rows, if any, occur at the bottom

(ii) each leading (non zero) entry in a row is in a column to the right of the leading entry of the row above it

ii) Def^n 1.25 - An echelon matrix E is said to be in reduced echelon form if, in addition,

(i) in each non-zero row, the leading entry is 1, and

(ii) in each col that contains the leading entry of a row, all other entries are zero

b) $A = \begin{pmatrix} -1 & -2 & -2 & -1 \\ 1 & 2 & 0 & 3 \\ 2 & 4 & 3 & 3 \\ 3 & 6 & 5 & 4 \end{pmatrix}$

$\xrightarrow{r_4 \to r_4 - r_3 + r_1} \begin{pmatrix} -1 & -2 & -2 & -1 \\ 1 & 2 & 0 & 3 \\ 2 & 4 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\xrightarrow[r_3 \to r_3 + 2r_1]{r_2 \to r_2 + r_1} \begin{pmatrix} -1 & -2 & -2 & -1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\xrightarrow{r_2 \to r_2 - 2r_3} \begin{pmatrix} -1 & -2 & -2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\xrightarrow[r_3 \to -r_3]{r_1 \to -r_1} \begin{pmatrix} 1 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\xrightarrow{r_1 \to r_1 - 2r_2} \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\xrightarrow{r_2 \leftrightarrow r_3} \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

c) A not invertible as A cannot be reduced to I_4 (rows not LI)

~~[illegible]~~

d) no, we've shown only two are LI by applying row reduction (row reduc is like simultaneous eqns)

we can build r_2, r_4 from r_1 and r_3

e) row space of A has basis

$\{(1,2,0,3),\ (0,0,1,-1)\} = \{a_1, a_2\}$

span + LI by row reduc

f) $B = \begin{pmatrix} 1 & 2 & 4 & -1 \\ 2 & 4 & 6 & 0 \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \end{pmatrix}$

~~$\to \begin{pmatrix} 1 & 2 & 4 & -1 \end{pmatrix}$~~

$r_1 = a_1 + 4a_2$

$r_2 = 2a_1 + 6a_2$

$\Rightarrow$ $\text{row}(B) \subseteq \text{row}(A)$

~~$\text{row}(A) \subseteq \text{row}(B)$~~

~~$\text{row}(A) \supseteq \text{row}(B)$~~

~~$a \in \text{row}A$~~

~~$a = \lambda_1 r_1 + \lambda_2 r_2$~~

r_1, r_2 LI as not a LC of each other

$\Rightarrow \dim(B) = \dim(A)$

$\Rightarrow \text{row}(A) = \text{row}(B)$ by Thm 3.45

$C = I_4$

$\dim(C) = 4$ $\quad \dim(A) = 2$

$\Rightarrow$ can't possibly have the same row space

g) rank(A) = # non zero rows in RREF of A
= 2 by ~~g~~ b

h) ~~W ⊆ V is a subsp~~ Defⁿ 2.11
A subspace W of V is a non-empty subset of V which itself forms a vec space under the same ops

i) E_1 = all Echelon 3x3 in $\mathbb{R}$
E_2 = reduced ech 3x3 in $\mathbb{R}$

$$E_1, E_2 \subseteq M_{3\times3}(\mathbb{R})$$

E_1 a subspace of $M_{3\times3}(\mathbb{R})$

* $E_1 \neq \emptyset$ $I_3 \in E_1$
* $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \in E_1$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix} \notin E_1$$ as non zero rows not at bottom

no, not closed under +

E_2 no

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \in E_2$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \notin E_2$$

leading entries $= 2 \neq 1$

no, not closed under +

2) a) V a VS   F a Field

i) LI

Defn 3.13 - $V_1 \dots V_n \in V$ are LI if the only soln to

$$\alpha V_1 + \alpha_2 V_2 + \dots + \alpha_n V_n = \underline{0} \qquad \alpha_i \in F$$

is $\alpha_1 = \alpha_2 = \dots = \alpha_n = 0$

ii) Defn 3.13 - $\{V_1 \dots V_n\} \subseteq V$ is a spanning set ~~fs~~ for V if $\forall v \in V$ $\exists$ $\alpha_1 \dots \alpha_n \in F$ st

$$V = \alpha_1 V_1 + \dots + \alpha_n V_n$$

iii) Defn 3.23 - A set $\beta = \{V_1 \dots V_n\}$ is a basis for V if they are both a spanning set for V and LI

b) i) $S_1 = \{(a, b, a+b, 1) \mid a, b \in \mathbb{R}\}$

no $(1,1,2,1), (0,0,0,1) \in S_1$

$(1,1,2,1) + (0,0,0,1)$

$= (1,1,2,2) \notin S_1$

(or no zero, zero in $\mathbb{R}^4$ is $(0,0,0,0)$)

ii) $S_2 = \{(a,b,c,d) \in \mathbb{R}^4 : a+b = c+d\}$

$= \{(a, b, c, a+b-c) \in \mathbb{R}^4\}$

(by subspace test)

$a = b = c = 0 \Rightarrow \underline{0} \in S_2 \Rightarrow S_2 \neq \emptyset$

$(a,b,c,a+b-c), (a',b',c',a'+b'-c') \in S_1$

$(a,b,c,a+b-c) + (a',b',c',a'+b'-c')$

$= ((a+a'), (b+b'), (c+c'), (a+a') + (b+b') - (c+c'))$

$\in S_1$

$\lambda(a, b, c, a+b-c)$

$= (\lambda a, \lambda b, \lambda c, \lambda(a+b-c))$

$= (\lambda a, \lambda b, \lambda c, (\lambda a) + (\lambda b) - (\lambda c))$

$\in S,$ $\Rightarrow$ ✓ by SS test

iii) $S_3 = \{(a, b, a+b, a^2) \mid a, b \in \mathbb{R}\}$

$(1, 1, 2, 1), (1, 1, 2, 1) \in S_3$

$(1, 1, 2, 1) + (1, 1, 2, 1) = (2, 2, 4, 2)$

$a = 2 \quad a^2 = 4 \neq 2$

$\Rightarrow \notin S_3$

b) $W = \text{span}\{(1, 0, 1, 0), (2, 3, 2, 2), (1, 1, 2, 1), (0, -2, 1, -1)\}$

$r_1 \quad r_2 \quad r_3 \quad r_4$

i) these span the set by defn

$\Rightarrow$ remains to check LI

$\Rightarrow$ put in matrix and peform row ops to check LI

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 2 & 3 & 2 & 2 \\ 1 & 1 & 2 & 1 \\ 0 & -2 & 1 & -1 \end{pmatrix} \xrightarrow[r_3 \to r_3 - r_1]{r_2 \to r_2 - 2r_1} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 3 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & -2 & 1 & -1 \end{pmatrix}$$

$$\xrightarrow{r_4 \to r_4 + r_2 - r_3} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 3 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\Rightarrow$ not LI as a zero row

r_4 can be built from r_1, r_2, r_3

ii) $\dim W < 4$ as atmost 3 LI vec in span
$\Rightarrow |\text{basis}| < 4$

$\Rightarrow$ this cannot be a basis as these are 4 LI vec

d) $W = \{(a, b, 0, 0) \mid a, b \in \mathbb{R}\}$

$(0,0,0,0) \in W \Rightarrow W \neq \emptyset$

$(a, b, 0, 0), (c, d, 0, 0) \in W$

$(a, b, 0, 0) + (c, d, 0, 0)$
$= ((a+c), (b+d), 0, 0) \in W$

$\lambda \in F$
$\lambda(a, b, 0, 0) = (\lambda a, \lambda b, 0, 0) \in W$
$\Rightarrow W$ a subspace by ss test

$(a, b, 0, 0) = a(1, 0, 0, 0) + b(0, 1, 0, 0)$
$\Rightarrow \{(1,0,0,0), (0,1,0,0)\} = \mathcal{B}$ span W

Clearly LI (standard basis subset)
$\Rightarrow \dim W = |\mathcal{B}| = 2$

e) V vs over F

$U, W \subseteq V$ subspaces

* $\underline{0} \in U, W \Rightarrow \underline{0} \in U \cap W$

$\Rightarrow U \cap W \neq \emptyset$

* $\underline{u}, \underline{v} \in U \cap W$

$\Rightarrow \underline{u}, \underline{v} \in U \qquad \underline{u}, \underline{v} \in W$

$\Rightarrow \underline{u} + \underline{v} \in U \qquad \underline{v} + \underline{v} \in W$ both ss

$\Rightarrow \underline{u} + \underline{v} \in U \cap W$

* $\underline{u} \in U \cap W \quad \lambda \in F$

$\Rightarrow \underline{u} \in U \qquad \underline{u} \in W$

$\Rightarrow \lambda \underline{u} \in U \qquad \lambda \underline{u} \in W$ both ss

$\Rightarrow \lambda \underline{u} \in U \cap W$

$\Rightarrow$ subspace by ss test

$U = \{(a,0,0,0) \mid a \in \mathbb{R}\}$

$W = \{(0,0,0,b) \mid b \in \mathbb{R}\}$

exclusive or

$U \cup W = \{(a,b,c,d) \mid b = c = 0,\ a \neq 0$ or $d \neq 0\}$

$(1,0,0,0) \in U \qquad (0,0,0,1) \in W$

$\Rightarrow (1,0,0,0), (0,0,0,1) \in U \cup W$

$(1,0,0,0) + (0,0,0,1)$

$= (1,0,0,1) \notin U \cup W$

as $\notin U$ last coord $\neq 0$

$\notin W$ 1st $\neq 0$

3)a)

i) defn 4.1 - V, W VS on same field F

$T: V \to W$ each $v \in V$ ass a vec

$T(v) \in W$ is a LT if

i) $T(u+v) = T(u) + T(v) \quad \forall u, v \in V$

ii) $T(\alpha v) = \alpha T(v) \quad \forall v \in V \quad \forall \alpha \in F$

ii) 4.2 - all vec V mapped to zero of W by T

$\ker T = \{v \in V \mid T(v) = \underline{0}\}$

iii) 4.18 - image of $T: V \to W$ lineal consists of all images $T(v)$ of vec in V under T

$\operatorname{im} T = T(V) = \{T(v) \mid v \in V\}$

b) $\ker T$

* $T(\underline{0}) = \underline{0}$

$T(0\underline{0}) = 0\,T(\underline{0}) = \underline{0}$

$\Rightarrow \underline{0} \in \ker T \Rightarrow \ker T \neq \emptyset$

* $u, v \in \ker T$

$T(u+v) \overset{LT}{=} T(u) + T(v)$

$= \underline{0} + \underline{0}$ $\quad \swarrow u, v \in \ker T$

$= \underline{0}$

$\Rightarrow u+v \in \ker T$

* $u \in \ker T \quad \lambda \in F$

$T(\lambda u) = \lambda T(u) \quad \leftarrow LT$

$= \lambda \underline{0}$

$= \underline{0}$

$\Rightarrow \lambda u \in \ker T$

$\Rightarrow$ subspace by SS test

c) i) $T: \mathbb{R}^4 \to \mathbb{R}^3$

$$T\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} x+y+z \\ x-t \\ y+z+t \end{pmatrix}$$

$$T\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$T\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$T\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$T\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$$\text{Mat}(T) = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

ii) perform row ops to find basis for row space of $\text{Mat}\,T^T$

= col space $\text{Mat}\,T$

= im of T

$$\mathrm{Mat}(T)^T = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & -1 & 1 \end{pmatrix}$$

$$\xrightarrow{r_3 \to r_3 - r_2} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

$$\xrightarrow{r_2 \to r_2 - r_1} \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

$$\xrightarrow{r_4 \to r_4 - r_2} \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{r_2 \to -r_2} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{r_1 \to r_1 - r_2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\Rightarrow$ Image $= \mathrm{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \right\}$

LI + spanning by row reduc

$\Rightarrow$ also a basis

$\cancel{T\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}}$

$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \in \ker T \iff T\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \underline{0}$

$\iff x+y+z=0$

$x-t=0 \implies x=t$

$y+z+t=0$

$y+z+t=0$

$t \cancel{= z+y}$

$z=-y-t=-y-x$

$\implies \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \in \ker T \iff \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} x \\ y \\ -y-x \\ x \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ -(y+x) \\ x \end{pmatrix} = x\begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} + y\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$

$\implies \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$ span the ker

clearly LI $\implies$ a basis

d) let $T: V \to W$ be a ~~LT~~ LT

Rank nullity;

$\text{Rank}(T) + \text{Nullity}(T) = \dim(V)$

e) $A = \{(x, 0, z, 0) \mid x, z \in \mathbb{R}\}$

i) $F, g : \mathbb{R}^4 \to \mathbb{R}^4$

$F : \mathbb{R}^4 \to \mathbb{R}^4$

$$F\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} x \\ 0 \\ z \\ 0 \end{pmatrix}$$

$$F\left(\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} + \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}\right) = F\begin{pmatrix} x+a \\ y+b \\ z+c \\ t+d \end{pmatrix}$$

$$= \begin{pmatrix} x+a \\ 0 \\ z+c \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} x \\ 0 \\ z \\ 0 \end{pmatrix} + \begin{pmatrix} a \\ 0 \\ c \\ 0 \end{pmatrix}$$

$$= F\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} + F\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$F\left(\lambda\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}\right) = F\begin{pmatrix} \lambda x \\ \lambda y \\ \lambda z \\ \lambda t \end{pmatrix}$$

$$= \begin{pmatrix} \lambda x \\ 0 \\ \lambda z \\ 0 \end{pmatrix}$$

$$= \lambda\begin{pmatrix} x \\ 0 \\ z \\ 0 \end{pmatrix}$$

$$= \lambda F\begin{pmatrix} x \\ 0 \\ z \\ 0 \end{pmatrix}$$

$\Rightarrow F$ linear

$$\operatorname{im} F = \left\{ \begin{pmatrix} x \\ 0 \\ z \\ 0 \end{pmatrix} \;\middle|\; x, z \in \mathbb{R} \right\}$$

$$= A$$

ii) $g: \mathbb{R}^4 \to \mathbb{R}^4$

$$g\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ y \\ 0 \\ t \end{pmatrix}$$

explain on day;
check linear

4)a) def^n 5.1

$A \in M_{n\times n}(F)$ $\quad v \in F^n$ $v \neq \underline{0}$ Evec if

$$Av = \lambda v$$

for $\lambda \in F$

λ = Eval of A

b) i) $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

$$\det(A - \lambda I) = \det \begin{pmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{pmatrix}$$

$$= (1-\lambda)\left[(1-\lambda)^2 - 0\right] + 0 - 1\left(0 - (1-\lambda)\right)$$

$$= (1-\lambda)\left[(1-\lambda)^2 + 1\right]$$

$$= (1-\lambda)\left[1 - 2\lambda + \lambda^2 + 1\right]$$

$$= (1-\lambda)\left[2 - 2\lambda + \lambda^2\right]$$

$\lambda = 1$

$\lambda^2 - 2\lambda + 2 = 0$

$$\lambda = \frac{2 \pm \sqrt{2^2 - 4(2)(1)}}{2}$$

$$= \frac{2 \pm \sqrt{4 - 8}}{2}$$

$$= \frac{2 \pm \sqrt{-4}}{2} = 1 \pm i$$

b) i) $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

$$\det(A - \lambda I) = \det\begin{pmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{pmatrix}$$

$$= (1-\lambda)\left[(1-\lambda)^2 - 0\right] - 0 + 1\left[0 - 1(1-\lambda)\right]$$

$$= (1-\lambda)(1-\lambda)^2 - (1-\lambda)$$

$$= (1-\lambda)\left[1 - 2\lambda + \lambda^2 - 1\right]$$

$$= (1-\lambda)(\lambda^2 - 2\lambda)$$

$$= (1-\lambda)\lambda(\lambda - 2)$$

$$\Rightarrow \lambda = 1, 0, 2$$

$\underline{\lambda = 1}$

$A\underline{v} = \underline{v}$

$\Rightarrow (A - I)\underline{v} = \underline{0}$

$\Rightarrow \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \underline{0}$

$\Rightarrow \begin{pmatrix} c \\ 0 \\ a \end{pmatrix} = \underline{0}$

$\Rightarrow c = a = 0$

$\Rightarrow \underline{v} \in \left\{ \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} \middle| \, b \in \mathbb{R} \right\}$ ie $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\underline{\lambda = 0}$

$A\underline{v} = 0\underline{v} = \underline{0}$

$\Rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} a+c \\ b \\ a+c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\Rightarrow b = 0 \quad a = -c$

$\Rightarrow \underline{v} \in \left\{ \begin{pmatrix} a \\ 0 \\ -a \end{pmatrix} \middle| \, a \in \mathbb{R} \right\}$ ie $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$\underline{\lambda = 2}$

$A\underline{v} = 2\underline{v}$

$\Rightarrow (A - 2\underline{I})\underline{v} = \underline{0}$

$\begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \underline{0}$

$\Rightarrow \begin{pmatrix} -a+c \\ -b \\ a-c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\Rightarrow a = c \quad b = 0$

$\Rightarrow \underline{v} \in \left\{ \begin{pmatrix} a \\ 0 \\ a \end{pmatrix} \middle| \, a \in \mathbb{R} \right\}$ ie $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

ii) P, D

$$P = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 1 \end{pmatrix} \qquad D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

c) i) λ Eval of A invert

Assume for # $\lambda = 0$

$\Rightarrow A\underline{v} = \lambda \underline{v}$

$\Rightarrow A\underline{v} = 0\underline{v} = \underline{0}$

$\Rightarrow A\underline{v} = \underline{0}$

A invert $\Rightarrow A^{-1}$ exists

$\Rightarrow A^{-1}A\underline{v} = A^{-1}\underline{0}$

$\Rightarrow I\underline{v} = \underline{0}$

$\Rightarrow \underline{v} = \underline{0}$

$\Rightarrow$ # by defn $\underline{v} \neq \underline{0}$

ii) $A\underline{v} = \lambda \underline{v}$

A invert $\Rightarrow A^{-1}$ exists

$\Rightarrow A^{-1}\cancel{A}\underline{v} = A^{-1}\lambda\underline{v}$

$\Rightarrow I\underline{v} = A^{-1}\lambda\underline{v}$

$\Rightarrow \underline{v} = \lambda A^{-1}\underline{v}$

$\Rightarrow \lambda^{-1}\underline{v} = A^{-1}\underline{v}$) allowable by (i)

$\Rightarrow A^{-1}\underline{v} = \lambda^{-1}\underline{v}$ $\Rightarrow$ yes!